Transition to strictly solitary motion in the Burridge-Knopoff model of multicontact friction

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We show that, in the continuous 1D Burridge-Knopoff model of multicontact friction, motion occurs via stick-slip sliding on a finite length rather than in avalanches, excluding the occurrence of self-organized criticality. We present strong numerical evidence that a transition from collective to strictly solitary motion occurs at a critical value of the interblock interactions. The solitary motion corresponds to successive stick-slip motion of one block between immobile neighbors, repeated periodically in time. This state persists also with open boundary conditions and moderate temperature.

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I. INTRODUCTION

Solid on solid sliding friction is often modeled by one-dimensional spring-block models, meant to represent very different situations. At the atomic scale, friction is well described by the Frenkel-Kontorova, or by the Frenkel-Kontorova-Tomlinson model [1], where the blocks represent individual atoms in interaction with a surface represented as a rigid periodic modulation. At much larger length scales, the Burridge-Knopoff (BK) model, illustrated in Fig. 1, is used to describe sliding tectonic plates. In the BK model, the interaction with the underlying surface is replaced by a phenomenological velocity dependent friction force with a static and a dynamic contribution. The dynamics of tectonic sliding is usually studied by assuming a dynamic friction force that weakens as a function of velocity [2].

In all these models where energy is slowly fed to the system by the moving plate, the dynamics is not uniform but dominated by fast dissipative events corresponding to stick–slip motion of the individual blocks. In their velocity weakening BK model, Carlson and Langer have shown [2] that avalanches of all sizes occur, with a power law size distribution compatible with the empirical Gutenberg-Richter law. This lack of an intrinsic length scale puts this deterministic continuous model into the larger class of systems which are said to display self-organized criticality (SOC), [3,4] a term introduced [3] to describe the behavior of discrete sandpile automata. Since the finding of Carlson and Langer, the BK model has been studied intensively in this context, particularly in the two-dimensional discretized version proposed by Olami, Feder, and Christensen [5] (OFC). However, several authors claim or suggest that the model does not display criticality [6–8]. It has even been conjectured that the asymptotic avalanche size distribution is dominated by avalanches of size one, the fraction of larger avalanches converging towards zero as the system size increases [9].

Here we study the multicontact friction variant of the BK model, proposed by Persson [10] to model macroscopic sliding systems in the boundary lubrication regime. The BK model of multicontact friction uses a viscous dynamic friction proportional to velocity, which, contrary to the velocity weakening earthquake models, effectively reduces the range of interactions of the blocks. This approach is justified by previous studies of the same author [11] showing that, at low velocity, a thin lubricant layer exhibits a distribution of pinned solid islands that fluidify and begin to slide when the applied force exceeds a threshold value and pin again as their velocity vanishes.

We find that, after an initial transient, the motion occurs as successive domino-like slipping events of limited size rather than in avalanches, thereby excluding the occurrence of SOC. At a critical value of the interblock interactions close to realistic values for sliding surfaces in the boundary lubrication regime [10], the system reaches a dynamic regime, that we call a solitary state, where the motion occurs via periodic step-like slipping events of single blocks. Surprisingly, the solitary state is not destroyed by open boundary conditions, contrary to the behavior of OFC models [12]. Also the solitary state is robust against small thermal fluctuations.

II. BK MODEL

The BK model of Fig. 1 consists of N blocks of mass m connected, at fixed distances D, to a plate moving at constant velocity v_s by springs of spring constant k_1, and to nearest-neighbor blocks by springs of spring constant k_2 and rest lengths D. The plate coordinate is $x=v_s t$, and q_i is the position of block i with respect to its initial equilibrium position $q_i(0)=0$. The force on a block at rest (i.e., $\dot{q}_i=0$) is

$$F_i = k_1(x-q_i) + k_2(q_{i+1}+q_{i-1}-2q_i). \qquad (1)$$

This force is balanced up to a threshold value F_s by the static friction force, so that a block remains motionless until it

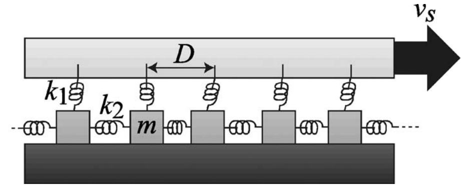

FIG. 1. Burridge-Knopoff (BK) model.

experiences a force $F_i \geqslant F_s$. Once in motion, a block is subjected to a viscous force $-2m\gamma\dot{q}_i$. If the block velocity $\dot{q}_i$ vanishes, the static friction force is reintroduced by setting the block velocity to zero if it changes sign. For this reason we always remain in the underdamped regime. The discontinuity of the friction force at $\dot{q}_i=0$ makes the system extremely nonlinear.

We introduce a dimensionless quantity characterizing the dynamic state of block i:

$$h_i \equiv \begin{cases} 0 & \text{if } \dot{q}_i = 0 \text{ (stick)} \\ 1 & \text{otherwise (slip)} \end{cases} . \tag{2}$$

We also introduce

$$\mathcal{H}_i(t) \equiv h_i(t)\left[h_{i+1}(t) + h_{i-1}(t)\right] \tag{3}$$

as the number (0, 1, 2) of neighbors slipping while block i is slipping. Note that $\mathcal{H}_i=0$ either when block i is at rest ($h_i=0$) or when block i is moving while both neighbors are at rest ($h_{i\pm1}=0, h_i=1$). Since the fraction of time a block is in motion can be quite small, it is useful to average Eq. (3) over a time τ around t

$$\langle \mathcal{H}_i(t)\rangle_\tau \equiv \frac{1}{\tau}\int_{t-\tau}^{t} \mathcal{H}_i(t')\mathrm{d}t', \tag{4}$$

yielding a continuous function, ranging between 0 and 2. By defining $\bar{h}(t)$ as the fraction of blocks moving at time t, the average over all moving blocks

$$\langle H(t)\rangle_\tau \equiv \begin{cases} \dfrac{1}{N}\displaystyle\sum_i^N \left\langle \dfrac{\mathcal{H}_i(t)}{\bar{h}(t)} \right\rangle_\tau, & \bar{h}(t) = \displaystyle\sum_i^N \dfrac{h_i}{N} \neq 0 \\ 0, & \bar{h}(t) = 0 \end{cases} , \tag{5}$$

constitutes an order parameter denoting if a system is either in solitary motion, i.e., $\langle H\rangle_\tau=0$, or in collective motion, $0<\langle H\rangle_\tau<2$.

The equations of motion are

$$m\ddot{q}_i = h_i[-2m\gamma\dot{q}_i + k_1(x-q_i) + k_2(q_{i+1}+q_{i-1}-2q_i)],$$

where

$$h_i(t+\mathrm{d}t) = \begin{cases} 0, & \dot{q}_i(t)\dot{q}_i(t+\mathrm{d}t) < 0 \\ 1, & F_i(t+\mathrm{d}t) \geqslant F_s \\ h_i(t), & \text{otherwise} \end{cases} , \tag{6}$$

with dt the time step of numerical integration. The equations of motion are made dimensionless by scaling time by $\sqrt{m/k_1}$, positions by F_s/k_1 and forces by F_s:

$$\ddot{q}_i = h_i\left[-2\tilde{\gamma}\dot{q}_i - \omega_0^2 q_i + \tilde{k}_2(q_{i+1}+q_{i-1}) + x\right] \equiv h_i\sigma_i, \tag{7}$$

with $\omega_0=\sqrt{1+2\tilde{k}_2}$ and σ_i denoting the total force on block i irrespective of its dynamic state h_i. Note that $\tilde{k}_2$ and $\tilde{\gamma}$ are in units of k_1 and $\sqrt{k_1/m}$, respectively, and that $\tilde{F}_s=1$. We will only consider dimensionless quantities, and will omit the tilde from now on.

The Eqs. of motion (7) are integrated by a fourth order Runge-Kutta algorithm with time step dt=0.005. The initial positions $q_i(0)$ are chosen from a uniform random distribution q= [−0.005,0.005]; furthermore $x(0)=0$ and $\dot{q}_i(0)=0$. We use periodic boundary conditions, unless specified otherwise. The width of the random distribution determines the duration of the transient collective stick–slip behavior. We consider a driving velocity $v_s=0.005$, which is low enough to be in the limit $v_s \ll \max(\dot{q}_i)$ characterizing typical tribological experiments.

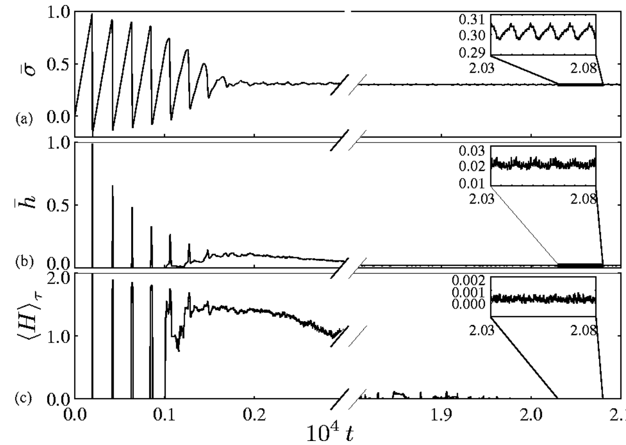

FIG. 2. Time dependence of (a) the average force $\bar{\sigma}$ (b) the fraction $\bar{h}$ of blocks moving and (c) the measure of collective behavior $\langle H\rangle_\tau$ for N=10 000, γ=0.5, k_2=1, τ=0.5. Panels (a) and (b) reproduce Fig. 4 of Ref. [10] extended to larger time. Notice in (c) the transition around $t\sim 20\,000$ to solitary motion, causing $\bar{\sigma}$ and $\bar{h}$ [see insets of panels (a) and (b)] to become periodic in time. Note that $\Delta t \gg \tau$ in this figure.

III. SOLITARY VERSUS COLLECTIVE MOTION

In Figs. 2(a) and 2(b) we show the average force $\bar{\sigma}$ and the fraction $\bar{h}$ of moving blocks as in Ref. [10] on a much longer timescale. The initial collective stick–slip behavior is due to the very narrow distribution of forces below F_s at $t=0$. At the first such collective event almost all blocks slip at the same time ($\bar{h}\approx 1$). As time progresses the distribution of forces $P(\sigma)$ widens and the number of blocks slipping at the same time decreases. After $t\sim 1000$, at any time a number of blocks is moving and, at $t\sim 1800$, the system is said to be in a steady state in Ref. [10].

In the steady state however, the fraction $\bar{h}$ of moving blocks keeps decreasing, indicating that the system is still equilibrating towards a more favorable state. Finally, at $t=20\,000$, we find that $\bar{\sigma}$ and $\bar{h}$ become periodic in time. It is shown in Fig. 2(c) that $\langle H\rangle_\tau\approx 0$ when the system becomes periodic. This indicates that blocks slip in a step-like fashion between immobile nearest neighbors ($h_{i\pm1}=0$), whence the name of *solitary motion*. Once this is the case for the motion of all blocks for longer than the interval between successive slips of the same block, the system is trapped in this solitary state and becomes periodic.

Analytical results give a rationale for this behavior. For solitary motion ($h_{i\pm1}=0$ when $h_i=1$) the equations of motion

(7) become decoupled, and the motion of a single block is that of a discontinuously driven, damped harmonic oscillator. For initial conditions $q_i(0)=\dot{q}_i(0)=0$ and $F_i(0)=F_s=1$ [i.e., $k_2(q_{i+1}+q_{i-1})+x=F_s$], and by assuming $v_s \ll \max(\dot{q}_i)$:

$$\ddot{q}_i + 2\gamma\dot{q}_i + \omega_0^2 q_i = F_s. \tag{8}$$

The solution of Eq. (8) for the underdamped case ($\gamma<\omega_0$)

$$q_i(t) = \frac{F_s}{\omega_0^2}\left\{1-\exp(-\gamma t)\left[\frac{\gamma}{\omega}\sin(\omega t)+\cos(\omega t)\right]\right\} \tag{9}$$

reaches zero velocity after a time

$$\delta t = \frac{\pi}{\omega}, \quad \text{with } \omega = \sqrt{\omega_0^2-\gamma^2}. \tag{10}$$

In a time δt the block travels a distance [13]

$$\Delta q = \frac{F_s}{\omega_0^2}[1+\exp(-\gamma\pi/\omega)]. \tag{11}$$

The interval Δt between consecutive slip events of the same block is given by

$$\Delta t = \Delta q / v_s, \tag{12}$$

because, although most of the time a block is not moving, its average velocity has to be equal to the plate velocity v_s. The fraction of time a block is moving, is simply the ratio of the duration of a slip event and the interval between them: $\bar{h} = \delta t/\Delta t$.

In the interval Δt between slip events, the force F_i acting on block i is slowly increased by the movement of the plate by an amount Δq ($k_1\Delta q$ in dimensional units), and by the sudden movement of both neighbors by an amount $2k_2\Delta q$. Therefore, the force directly after the slip event is $F_{\min}=1-(1+2k_2)\Delta q$ (since $F_s=1$). We can identify three ranges of the forces acting on a block:

$$\begin{aligned} 1-(1+2k_2)\Delta q &\leq F_i \leq 1-2k_2\Delta q \quad \text{low} \\ 1-(1+k_2)\Delta q &\leq F_i \leq 1-k_2\Delta q \quad \text{medium.} \\ 1-\Delta q &\leq F_i \leq 1 \quad \text{high} \end{aligned}$$

A block is in the low force range after it has slipped, moves to the medium range when one neighbor has slipped, and to the high range when both neighbors have slipped. Movement within each range is caused by the slow motion of the plate.

Figure 3(a) shows a snapshot of the forces on part of the chain in the solitary regime. Peaks of only one block are present in the lower and higher force range, separated by slanted lines in the medium force range where most of the blocks reside. In Fig. 3(b) we show the distribution of forces $P(\sigma)$ around the time of the snapshot of Fig. 3(a). $P(\sigma)$ is peaked at $\sigma=1$, and $\sigma=1-(1+2k_2)\Delta q$ due to the predominance of lines with a small slope.

The distribution of forces $P(\sigma)$ in the solitary state shown in Fig. 3(b) is highly symmetric, hence its mean $\bar{\sigma}$ can be approximated by the center of the distribution:

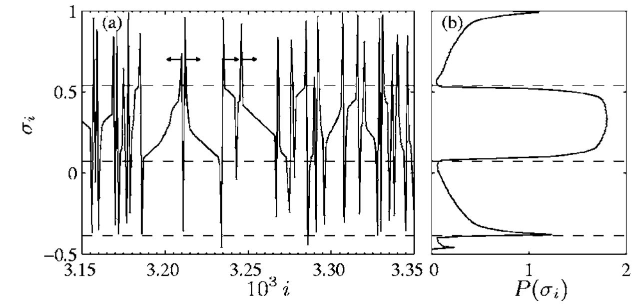

FIG. 3. (a) Forces σ_i for part of a system in a solitary state, with $k_2=1$, $\gamma=0.5$, $N=10\,000$, and (b) the distribution of forces measured over $N=10\,000$ blocks over a period of 3000 time steps. Dashed lines from top to bottom indicate $\sigma=1-\Delta q$, $\sigma=1-(1+k_2)\Delta q$ and $\sigma=1-(1+2k_2)\Delta q$, respectively. Arrows indicate which blocks will be in the high force range next. The peak in the distribution at $\sigma<1-(1+2k_2)\Delta q$ is caused by moving blocks, and vanishes for $v_s=0$.

$$\bar{\sigma} \approx \frac{1-[1-(1+2k_2)\Delta q]}{2} = \frac{1}{2}[1-\exp(-\gamma\pi/\omega)], \tag{13}$$

where we have made use of Eq. (11) for Δq. The friction force measured in experiments is the lateral force acting on the support

$$f = \sum_{i=1}^{N}(q_i - x) \approx -N\bar{\sigma}, \tag{14}$$

where we have assumed in Eq. (7) that $\Sigma_i(q_{i\pm1}-q_i)\approx 0$ and $\Sigma_i\dot{q}_i \approx N v_s \approx 0$. Since the kinetic friction force Eq. (14) is normalized by the static friction force F_s, this result implies that the ratio of the kinetic to the static friction force in the solitary state can be used to extract, from experiments, the ratio γ/ω characterizing the sliding system.

The analysis of the behavior of the forces σ_i in the solitary state, leads us to define a typical length scale in the system. We find that solitary motion requires two consecutive blocks in the high force range to be separated by an arbitrary number of blocks in the medium force range, and by exactly one block in the low force range. The blocks in the medium force range are arranged in monotonically increasing or decreasing slanted lines that will reach the high energy region one after the other. However, since in the time δt it takes block i to slip, the upper surface travels a distance $v_s\delta t$, the absolute slope of the lines is constrained by

$$\left|\frac{d\sigma}{di}\right| \geq v_s\delta t. \tag{15}$$

Since, in the strictly solitary state, each slanted line must start and end in the medium force zone $\Delta\sigma=\Delta q$ wide, the minimum slope also limits the number of blocks along the line to

$$N_{\text{line}} = \left|\frac{di}{d\sigma}\right|\Delta\sigma \leq \frac{\Delta q}{\delta t v_s} = \frac{\Delta t}{\delta t}. \tag{16}$$

The finite duration δt of a slip event introduces a typical length scale, contrary to systems displaying SOC. Strictly

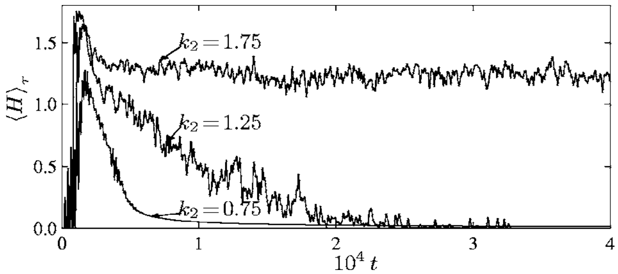

FIG. 4. Order parameter for collective motion $\langle H \rangle_\tau$ as a function of time, $\tau = \Delta t(k_2)$ [Eq. (12)], for three values of k_2 above and below $k_2^c \sim 1.5(\gamma = 0.5, N = 10\,000)$. At $k_2 = 0.75$ the decrease of $\langle H \rangle_\tau$ is smooth, whereas at $k_2 = 1.25$ $\langle H \rangle_\tau$ also decreases to zero, but only after several attempts. At $k_2 = 1.75$ $\langle H \rangle_\tau$ tends to a constant finite value.

speaking, in a continuous model, the size of an avalanche is given by the number of blocks performing simultaneous motion. By this definition, in a system in solitary motion, all avalanches are of size one. However, sequences up to N_{line} of size one avalanches can and do occur.

Next, we show in Fig. 4 the time evolution of $\langle H \rangle_\tau$ for three values of k_2. We find that a transition occurs at a critical relative value of the spring constant $k_2^c \sim 1.5$. Below k_2^c, $\langle H \rangle_\tau$ smoothly decreases to zero, signaling the occurrence of the solitary state, whereas, above k_2^c, $\langle H \rangle_\tau$ reaches a constant finite value. An estimate of parameters for realistic sliding lubricated surfaces [10] gives $k_2 \sim 1$. For values of k_2 just below k_2^c there is an initial, relatively smooth decrease of $\langle H \rangle_\tau$, but the solitary state is reached only after many attempts. This process is shown in the left panel of Fig. 5 where we show a gray scale map of the order parameter $\langle H \rangle_\tau$ for k_2 below and above k_2^c. The initial uniform band corresponds to collective stick slip motion (see Fig. 2). This behavior is followed by a very short period of almost uniform motion with velocity v_s, appearing as black regions in the figure. Notice that uniform motion has been shown to be unstable [2] for models with a velocity weakening friction force. Afterwards, for $k_2 < k_2^c$, domains of solitary motion of different sizes grow and shrink, until finally the complete system is in a solitary state. For $k_2 > k_2^c$, the system remains

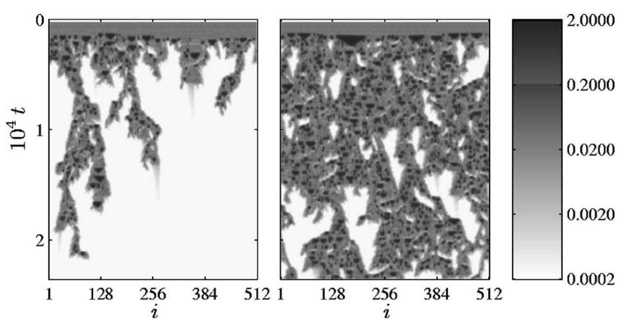

FIG. 5. $\langle \mathcal{H}_i \rangle_\tau$ as a function of time and block number i for $\gamma = 0.5, N = 512, \tau = \Delta t(k_2 = 0)$. In the left panel $k_2 = 1.25 < k_2^c$ and in the right panel $k_2 = 1.75 > k_2^c$. The logarithmic color coding scheme is given to the right. White areas are in solitary motion. Note that also above k_2^c the solitary state appears, but does not extend to the whole system.

in a collective state, characterized by the fact that neighboring blocks move simultaneously for part of their movement, much like domino topplings. However, large patches of solitary motion that expand and disappear are also present. We expect the probability that, for $k_2 > k_2^c$, a patch of solitary motion extends to the whole system to vanish for $N \to \infty$.

There are several indications that the transition to solitary motion is of first order. For a given system size, increasing k_2 towards k_2^c increases the time needed to reach the solitary state, but does not change qualitatively the shape of the curve shown in Fig. 4 for $k_2 = 1.25$. Moreover, once the solitary state is reached, if k_2 is increased in small steps above k_2^c the system readjusts to remain in a now metastable solitary state, in analogy to overheating a system above T_c. We wish to underline however, that the evidence for a sharp transition is only numerical.

IV. STABILITY OF THE SOLITARY STATE

Next we study the stability of the solitary state for small perturbations caused by thermal fluctuations. Due to these fluctuations a block may temporarily obtain enough energy to slip, even though it is experiencing a force smaller than the static friction force. Following Persson [10] we can define an energy barrier for block i as

$$\Delta E_i = U(F_s, q_{i\pm1}, x) - U_i(F_i, q_{i\pm1}, x), \tag{17}$$

where $U_i(F_i, q_{i\pm1}, x)$ is the potential energy of block i, and $U(F_s, q_{i\pm1}, x)$ is the potential energy of the same block, moved to where it would experience the static friction force $F_s = 1$, while keeping the position of the neighboring blocks and of the plate fixed. The potential energy U_i is given by

$$\begin{aligned} U_i &= \frac{1}{2}(x - q_i)^2 + \frac{k_2}{2}(q_{i+1} - q_i)^2 + \frac{k_2}{2}(q_{i-1} - q_i)^2 \\ &= g(q_{i\pm1}, x) + \frac{F_i^2}{2\omega_0^2}. \end{aligned} \tag{18}$$

Since $g(q_{i\pm1}, x)$ does not depend on F_i, Eqs. (17) and (18) give

$$\Delta E_i = \Delta E_{\max}(1 - F_i^2), \quad \text{with } \Delta E_{\max} = 1/2\omega_0^2. \tag{19}$$

The probability that block i slips (i.e., overcomes the energy barrier) within a time dt is assumed to be

$$P_i(\mathrm{d}t) = \nu \exp(-\Delta E_i / k_B T)\mathrm{d}t, \tag{20}$$

where ν is an attempt frequency, T the temperature, and k_B the Boltzmann constant. In practice, finite temperature is simulated by drawing a random number $r_i = [0,1]$ at each integration step for each block, and if $r_i < P_i(\mathrm{d}t)$, where dt is the integration time step size, the static friction force is decreased to zero by setting $h_i = 1$.

In Fig. 6 the time dependence of the order parameter for collective motion $\langle H \rangle_\tau$ is shown at different temperatures for $k_2 = 1$ where at $T = 0$ the solitary state is stable. For low temperatures the order parameter goes to zero in a way similar to the zero temperature case, although small fluctuations do occur. These fluctuations grow with increasing temperature, un-

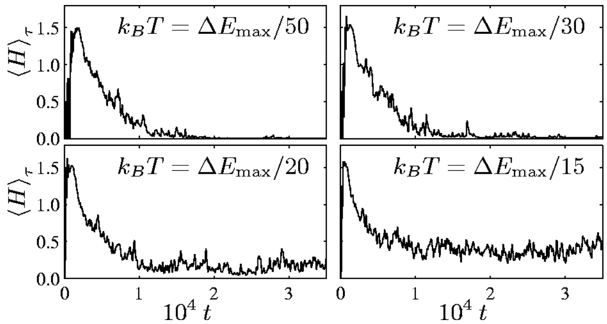

FIG. 6. Order parameter for collective motion $\langle H\rangle_\tau$ as a function of time for different temperatures, and for $\tau=\Delta t, k_2=1, \gamma=0.5$, and $N=10\,000$. The solitary state is stable at least up to $k_BT=\Delta E_{\max}/30$.

til the system cannot maintain the solitary state. In Fig. 7 we show a gray scale map of the order parameter per block i, as a function of time. At low temperature the system evolves towards the solitary state in much the same way as in the zero temperature case (compare with Fig. 5). Collective motion occurs only very locally, very weakly, and often only involves the direct neighbors of the blocks that were thermally excited. At higher temperatures, larger patches of collective motion appear, without ever extending to the whole system.

We recall that once every block in the system is moving in a solitary fashion for longer than the time between two slips Δt, the complete system is trapped in the solitary state. Since each block slips the same distance Δq, the system becomes periodic with a period Δt. A finite temperature gives rise to a finite probability for a block to slip at a force $F_i < F_s$, and since the distance a block slips is proportional to the force

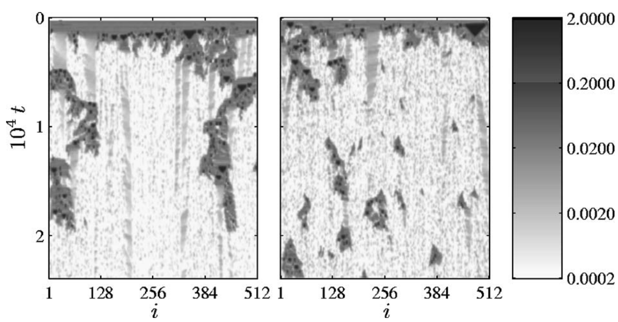

FIG. 7. $\langle \mathcal{H}_i\rangle_\tau$ for the parameters of Fig. 5 and $k_2=1$, at $k_BT=E_{\max}/30$ (left) and $k_BT=E_{\max}/15$ (right). In the low temperature case (left) small fluctuations only very briefly and locally amount to collective motion. The light gray vertical bands are caused by series of blocks moving with very small overlap in time. At higher temperature (right) thermal events may lead to large patches of collective motion, that however do not extend to the whole system.

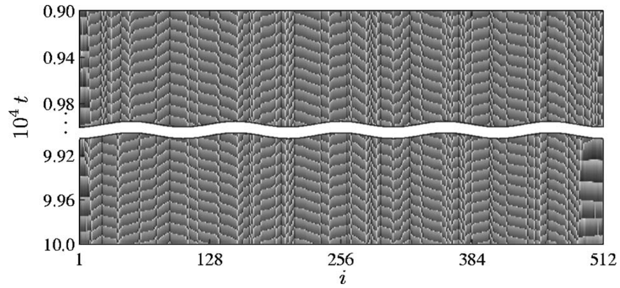

FIG. 8. Force $\sigma_i(t)$ in a system of $N=512$ blocks with $\gamma=0.5$ and $k_2=1$, with open boundary conditions. Black is $\sigma_i(t)=1$ and white is $\sigma_i(t)=1-(1+2k_2)\Delta q$. Note that the boundary conditions do not change the solitary state of the bulk $(60<i<470)$ even after $t=90\,000$ time steps $(\sim 10^3\Delta t)$. Also note the difference in interval Δt between solitary (bulk) and collective (edges) slip events.

acting on the block at the moment it slips [Eq. (11)], a thermally induced slip event breaks the perfect periodicity of the solitary state. However, as clearly shown in Fig. 7 the nature of the motion is not drastically different from strictly solitary motion.

Lastly in Fig. 8 we show that the solitary state is not destroyed by open boundary conditions. One can recognize the region of collective motion at the edges, because the interval between successive slip events of the same block is larger than in the solitary state. These regions of collective, nonperiodic motion appear, expand, and shrink at the boundary, but do not extend to the interior of the sample. This is remarkable because open boundary conditions are expected to destroy simple periodic states [12].

V. SUMMARY AND CONCLUSIONS

In summary we have shown that the motion in the continuous BK model with viscous friction at low driving velocities occurs in domains of finite size presenting an intrinsic length scale, thereby excluding the occurrence of SOC. Below a critical value of the interblock interaction the system evolves to a strictly solitary, periodic state with successive slipping of individual blocks among immobile neighbors. The solitary state is stable against small thermal fluctuations and open boundary conditions. In the range of parameters estimated to describe actual sliding systems [10], this model predicts strictly solitary motion, for which Eq. (14) can be used to measure the damping and stiffness of the sliding system.

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